Consider the region bounded by $y=(x-2)^2$ and $y=6-3x$.

$(x-2)^2 = 6-3x$

$x^2-4x+4 = 6-3x$

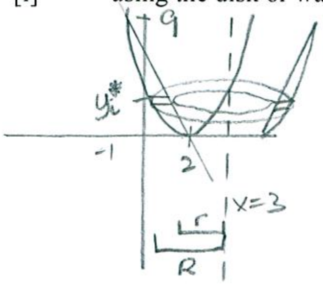
$x^2-x-2=0 \longrightarrow x = 2, -1$

$y = 0, 9$

SCORE: _____ / 15 PTS

[a] If the region is revolved around the line $x=3$,
write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid

[i] using the disk or washer method

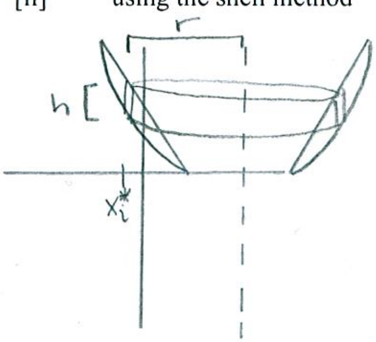


$$\underbrace{\pi}_{1}\int_{\underbrace{0}_{1}}^{\overbrace{9}^{1}} \Big(\underbrace{\big(3-(2-\sqrt{y})\big)^2}_{1} - \underbrace{\left(3-\tfrac{6-y}{3}\right)^2}_{1} \Big) \underbrace{dy}_{\frac{1}{2}}$$

$y=(x-2)^2 \longrightarrow x = 2 \pm \sqrt{y}$

$y = 6-3x \longrightarrow x = \frac{6-y}{3}$

[ii] using the shell method



$$\underbrace{2\pi}_{1}\int_{\underbrace{-1}_{1}}^{\overbrace{2}^{1}} \underbrace{(3-x)}_{1}\underbrace{\big(6-3x-(x-2)^2\big)}_{1}\underbrace{dx}_{\frac{1}{2}}$$

[b] Suppose the region is the base of a solid. Cross sections perpendicular to the $x-$axis are semicircles.
Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid.

$$\underbrace{\frac{\pi}{8}}_{1\frac{1}{2}}\int_{\underbrace{-1}_{\frac{1}{2}}}^{\overbrace{2}^{\frac{1}{2}}} \underbrace{\big(6-3x-(x-2)^2\big)^2}_{1}\underbrace{dx}_{\frac{1}{2}}$$

A solid with volume $\pi\int_0^1 ((3+\tan y)^2 - (3+\frac{y}{2})^2)\,dy$ is created by revolving a region around an axis of revolution. Sketch the region, and find the equations of its boundaries and the axis of revolution. Label all important points on the axes. **SCORE: _____ / 4 PTS**

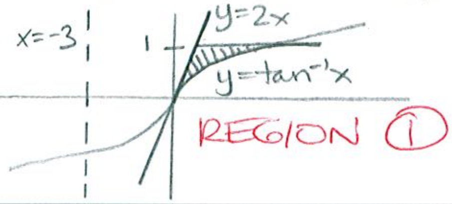
$\pi\int(R^2-r^2)\,dy \rightarrow$ HORIZONTAL CUT, WASHER METHOD, VERTICAL AXIS $(x=\#)$

$R = 3+\tan y = \tan y - (-3)$ AXIS $x=-3$ 1

$r = 3+\frac{y}{2} = \frac{y}{2} - (-3)$ BOUNDARIES $x = \tan y \rightarrow y = \tan^{-1}x$ 1

$x = \frac{y}{2} \rightarrow y = 2x$ 1

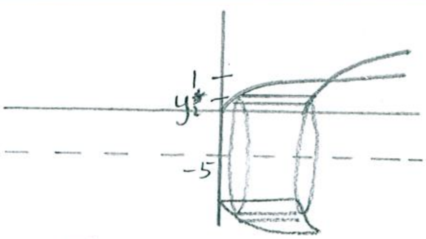
$y \in [0,1]$ (LIMITS OF $\int$)



The region bounded by $y=\frac{\sqrt{x}}{2}$, $y=\sqrt{x-3}$ and $y=0$ is revolved around the line $y=-5$. **SCORE: _____ / 5 PTS**

Write, **BUT DO NOT EVALUATE**, a **SINGLE** integral for the volume of the solid.

Note: No credit if your answer involves a sum or difference of two or more integrals.



$2\pi\int_0^1 (y - {-5})(y^2+3-4y^2)\,dy$

$= 2\pi \int_0^1 (y+5)(3-3y^2)\,dy$ 1 1 1/2 1 1 1/2

$\frac{\sqrt{x}}{2} = \sqrt{x-3}$

$\frac{x}{4} = x-3$

$x = 4x-12$

$x=4 \rightarrow y=1$

$y=\frac{\sqrt{x}}{2} \rightarrow x=(2y)^2=4y^2$

$y=\sqrt{x-3} \rightarrow x=y^2+3$

Find the area between the curves $y=3x^2-6x$ and $y=2x+3$ over the interval $1\le x\le 4$. **SCORE: _____ / 6 PTS**

$\int_1^4 |3x^2-6x-(2x+3)|\,dx$

$= \int_1^4 |3x^2-8x-3|\,dx$ 1

$= \int_1^3 -(3x^2-8x-3)\,dx + \int_3^4 (3x^2-8x-3)\,dx$ 1

$= -(x^3-4x^2-3x)\Big|_1^3 + (x^3-4x^2-3x)\Big|_3^4$ 1 1/2

$= -[(27-1)-4(9-1)-3(3-1)] + (64-27)-4(16-9)-3(4-3)$

$= -(26-32-6)+(37-28-3) = 12 + 6 = 18$ 1 1 1/2

$3x^2-8x-3 = (3x+1)(x-3)$

$= 0 \text{ @ } x = -\frac{1}{3}, 3$

$3x^2-8x-3 < 0$ ON $[-\frac{1}{3}, 3]$

≥ 0 ELSEWHERE